

SUMMER SCHOOL *Rigidity and group actions*
Institut de Mathématiques de Jussieu, 13-28 juin 2013

ABTRACTS

RIGIDITY OF GROUP ACTIONS : AROUND THE ZIMMER PROGRAM

David Fisher

This course will begin with a survey of *Zimmer's program* to classify actions of “large” groups on compact manifolds. There will be some emphasis on examples that show the limits of the program. Later lectures will focus on some particular aspects of this program, such as local rigidity of affine actions, actions of random groups and global rigidity of Anosov actions on nilmanifolds.”

LATTICES IN LIE GROUPS

Alexander Gorodnik

This minicourse will provide an introduction to the theory of lattices in Lie groups.

Lattices appear naturally in geometry as fundamental groups of locally symmetric spaces, in number theory as arithmetic groups, and are also of great interest from the point of view of dynamical systems because they exhibit remarkable rigidity properties.

We start by explaining geometric and arithmetic constructions of lattices.

After discussing basic properties of lattices, I will present some of the classical results about their structure, such as finite presentability, existence of unipotent and higher-rank abelian subgroups, existence of torsion-free subgroups, etc.

In the second part of the course, we discuss basic rigidity properties of lattices. In particular, we investigate existence of deformations and whether isomorphisms between lattices extend to the ambient Lie groups giving a sketch of a proof of the Mostow rigidity theorem.

GLOBAL RIGIDITY OF HIGHER RANK LATTICE ACTIONS.

Federico Rodriguez Hertz

In this course we shall present some recent classification results in the theory of measure preserving smooth lattice actions on manifolds. The theme of the course is the following :

Conjecture : If Γ is a lattice in a higher rank semisimple Lie group without compact factors and $\alpha : \Gamma \rightarrow \text{Diff}(M)$ is a smooth action preserving a measure μ then α is measurably equivalent to an algebraic action.

In this lecture sequence we plan to develop the tools needed to prove this Conjecture in the case the action has some weak hyperbolic property. We will try to restrict our attention to simple settings.

The lectures plan is the following :

- 1) Model actions : affine actions on tori.
- 2) Homotopy data of actions on tori.
- 3) Lifts and semiconjugacies.
- 4) Rigidity of derivative cocycle
- 5) Non-stationary normal forms.

- 6) Smoothness of conditional measures
- 7) Smoothness of invariant measures
- 8) Rank $n - 1$ in dimension n (general case)

Requirements : Basic ergodic theory and some knowledge of smooth ergodic theory and Pesin theory are welcomed.

AN INVITATION TO VON NEUMANN ALGEBRAS

Cyril Houdayer

This series of lectures will be devoted to introducing several concepts from the theory of finite von Neumann algebras. First, we will review some basic results from the theory of II_1 factors and study Murray-von Neumann's group measure space construction. Then, we will introduce several tools which play an important role in the structure and classification of II_1 factors : these include for instance Connes's theory of bimodules, Jones's basic construction and Popa's intertwining techniques. Then we will prove Connes's characterization of amenability for II_1 factors. Finally, we will show that the free group factors satisfy Haagerup's complete metric approximation property.

RIGIDITY FOR STABILIZERS OF MEASURE PRESERVING ACTIONS

Jesse Peterson

A strong generalization of the Margulis Normal Subgroup Theorem, due to Stuck and Zimmer, states that any properly ergodic probability measure-preserving action of an irreducible lattice in a center-free semisimple Lie group with all simple factors of higher-rank is essentially free. In my lecture series I will present a similar result, based on joint work with Darren Creutz, for lattices (and their commensurators) in semi-simple groups which are not connected. Along the way I will also describe how the techniques that arise are related to other notions such as amenability, property (T), invariant random subgroups, relative profinite completions, characters of representations, and von Neumann algebras.

 II_1 FACTORS WITH A UNIQUE CARTAN DECOMPOSITION**Sorin Popa and Stefaan Vaes**

Abstract : We provide an overview of the recent progress in the uniqueness and classification of Cartan subalgebras in II_1 factors. This will include a proof of the theorem that for any free ergodic probability measure preserving action of a free group, the associated group measure space II_1 factor has a unique Cartan subalgebra, up to unitary conjugacy. We will deduce that group measure space II_1 factors arising from actions of free groups with a different number of generators are never isomorphic.

Talk 1, Monday, 9h00-10h30.

- An introduction to deformation/rigidity theory for II_1 factors.
- An overview of recent classification and structure results for II_1 factors.

Talk 2, Tuesday, 14h15-15h15.

- Statement of our 2011 theorem : for all free ergodic probability measure preserving actions on (X, μ) of the free group \mathbb{F}_n , the II_1 factor $L^\infty(X) \rtimes \mathbb{F}_n$ has a unique Cartan subalgebra, up to unitary conjugacy.
- Structure of the proof.
- Intertwining-by-bimodules, a reminder from Cyril Houdayer's lectures.
- Amenability and relative amenability for von Neumann algebras.

Talk 3, Wednesday, 9h00-10h30.

- The complete metric approximation property (CMAP) for the free group \mathbb{F}_n , a reminder from Cyril Houdayer's lectures.
- Part 1 of the proof, using the CMAP property.

Talk 4, Thursday, 14h15-15h45. The role of the Akemann-Ostrand property.

- The free group \mathbb{F}_n is bi-exact, i.e. belongs to Ozawa's class \mathcal{S} .
- Part 2 of the proof, using bi-exactness.
- Conclusion : uniqueness of Cartan subalgebras holds for all crossed products by nonamenable, weakly amenable, bi-exact groups.

Problem session : Thursday, starting 16h15. An overview of open problems in the theory of II_1 factors.

DYNAMICAL SYSTEM IN HOMOGENEOUS SPACES

Jean-François Quint

The lectures will tend to present basic objects appearing in dynamical system in homogeneous spaces together with fundamentals results. This will entail : examples of construction of lattices in semi-simple Lie groups, mixing theorem of Howe-Moore and recurrence phenomenon of Dani and Margulis. Finally, particular examples of Ratner theorem will be proved.

Requirements : basic notions of Lie group theory and dynamical system.

ANALYTIC PROPERTIES OF DISCRETE GROUPS.

Romain Tessera

This series of lectures will treat several important analytic notions attached to (mainly infinite) groups. First, we shall introduce the notion of amenability in the sense of Von Neumann. This part of the course will culminate with the proof of Følner's characterization of amenability. After some reviews on unitary representations, we shall introduce two important and antagonist analytic properties : Kazhdan's property T and Haagerup's property. Various characterizations of these properties will be given.